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ABSTRACT

What should constitute a preschool mathematics program for children ages 3 and a half to 5 years? What mathematical ideas could youngsters explore? This paper discusses six broad mathematical concepts for investigation by preschoolers: number, space and location, shape, patterns, measurement, and permutations. For each big idea, activities are presented that illustrate aspects not traditionally offered to preschoolers. These activities are designed to enlarge children's understanding of mathematics and stimulate them to make conjectures, formulate their own problems, make decisions and predictions, discover ways to keep records of their experiments, and talk about their observations. There is no specific age designated for each activity. Following an exploration of these mathematical concepts and activities, the paper discusses the nature and importance of representation and communication in mathematics learning, and the role of the teacher. The paper concludes by noting that in order to facilitate young children's mathematical development, the preschool mathematics curriculum should be changed from a collection of unrelated activities to a cohesive development of the important ideas of number, space and location, shape, patterns, measurement, and permutations. These explorations should be designed to relate to children's daily experiences and current interests. (Contains 13 references.) (EV)

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READY TO LEARN: DEVELOPING YOUNG CHILDREN'S MATHEMATICAL POWERS

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Serena's Problem

The date, June 2. Serena, age 4 years 11 months, some classmates and her teacher, Ms. Clare, were sitting in a circle talking about the kindergarten adventure that lay ahead the following September. As the discussion ensued, Ms. Clare announced that the first day of kindergarten was September 4 which was "very soon!" "How many days?" asked Serena. "Do you mean how many days 'til kindergarten starts?" asked Ms. Clare. Serena nodded. As Ms. Clare searched for a calendar which she could not locate, she replied, "That's a wonderful question. How can we figure it out?" "I can count the days," replied Serena. The other children concurred; counting was a grand plan. Ms. Clare wrote the dates, June 2 and September 4 on the chalkboard.

Ms. Clare wrote the names of the months on the board, beginning with June and ending with September. By chiming in with Ms. Clare and repeating the familiar rhyme about days in the months ("Thirty days has September,..."), the children were able to tell Ms. Clare the number of days in each month, which she duly recorded next to the corresponding month. Just as Ms. Clare was returning to her seat, Jessie exclaimed, "That's 4." "What's 4?" asked Ms. Clare "September is 4." stated Jessie. Ms. Clare erased the 30, recorded 4 for the number of days in September. "Fix June!" exclaimed Serena. "June?" questioned Ms Clare. Serena jumped up, went to the chalkboard, pointed to the date, June 2, raised two fingers, counted silently, and stated, "See. There's 2." "How should we fix June?" asked Ms. Clare. After a number of theories were offered, Serena counted backward from 30, announcing the two numbers "30, 29", and directed Ms. Clare to record '29' next to June. Timothy did not agree. He began counting forward by ones from 1 and noted that there was only one number, not two, after "29". But Serena reigned supreme, and 29 was recorded for June. Ms. Clare then showed the children how to use a calculator to find the total number of days.

The next day Ms. Clare showed the children a calendar and suggested that they "check their answer" from the previous day. Using the calendar, the children counted the days and included both June 2 and September 4 in their count. Thus their answers with and without the calendar did not jibe! This resulted in another rich mathematical discussion about between-ness.

When educators and politicians talk about preschool as that time when we have to "get children ready to learn," I think of Serena's problem. Serena and her preschool peers posed a complex problem; designed a solution plan; used various counting strategies, counting back from 30, and counting forward into the 90's; interpreted information in a list and a calendar; formulated hypotheses; argued for their solutions; and persisted until they solved the problem. These children weren't getting ready to learn. They were learning. They were thinking mathematically!

Numerous studies have demonstrated that young children's thinking is far more complex than previously imagined (Flavell 1982; Siegler 1996), and that their knowledge of mathematical ideas is more robust than expected (Geary 1994; Ginsburg et al. 1997; Liben & Yekel 1996; Nunes & Bryant, 1996). Despite these findings, those few preschool mathematics programs already in existence are limited in their visions of what is possible for young children to explore. Most focus on only a few mathematical topics and the activities are often unrelated to one another and not sequenced by complexity. From listening to Serena and her classmates, it is clear that young children are capable of exploring more mathematical ideas and in greater depth than usually expected. And, they are eager to do so.

What should constitute a preschool mathematics program for children ages 3 1/2 to 5 years? What mathematical ideas could youngsters explore? In the discussion that follows, five big mathematical ideas for investigation by preschoolers, including number, space and location, shape, patterns, measurement, and permutations, are described. This is followed by a discussion of the nature and importance of representation and communication, and the role of the teacher.

For each big idea, activities are presented that illustrate unique aspects of the idea that are not traditionally offered to preschoolers, but which can enlarge their understanding of mathematics and

stimulate them to make conjectures, formulate their own problems, make decisions and predictions, discover ways to keep records of their experiments, and talk about their observations. There is no specific age designated for each activity. The great variability in children's thinking and strategies for attacking and solving problems would make such designations invalid (Siegler 1996). Ms. Clare and Ms. Abby are the fictitious names of two superb preschool teachers.

The Big Ideas

Number. A great many concepts related to number are available for young children to explore, including the uses of numbers to label (home address, telephone number), to tell how many (5 stickers) , to tell which one (the second person in line), to describe measurements (4 footsteps long); the construction of large numbers (place value); the comparisons of sets of objects (one-to-one correspondence; subset relationships); the mathematical operations (putting together, taking apart, and grouping); and the properties of whole number operations (the zero property of addition and subtraction, the property of commutativity). Most preschool programs focus on one-to-one-correspondence, the counting sequence, the use of counting to tell how many objects in a set, and the combining and partitioning of sets in preparation for addition and subtraction. Generally, the magnitude of the numbers is restricted in counting to 20 or less, to comparing sets of fewer than 10 objects, and to adding and subtracting with numbers less than 5. Large numbers and the number, zero, are rarely if ever explored. Yet, children are fascinated by these key number concepts and are ready to learn about them, as may be seen in the two activities that follow.

Zero, the Empty Bag! To help children understand that the number of objects in a bag is not related to the physical characteristics of the objects (e.g., size, shape, color, texture) nor to their position, Ms. Abby had her preschoolers fill plastic bags with 1, 2 or 3 objects. The objects came from a junk collection of small objects that the children had collected earlier in the year. Once the bags were filled, they were tied with twist ties and placed in number bins; each bin was labeled with one of the numerals, 1, 2, 3. Each day for several days, Ms. Abby had the children select and open two bags, and, with her assistance, compare their contents using one-to-one matching to tell which

bag "has more," "has less," or if the bags have "the same number" of objects. After the contents were compared, the children replaced objects in bags and bags in bins.

A couple of weeks later, after introducing children to the numbers, 4, 5, and 6, Ms. Abby presented the empty bag and its corresponding number, zero. Several bags were twist-tied and placed in the '0' bin. Thereafter, the children loved the idea of comparing bags of objects with the zero bag. They always knew which bag held more! When the children began to explore the putting together of sets to make other sets, they used these same bags, and were delighted with the idea that when you combined zero with any other number, you always get the other number (the zero property of addition) !

Big Numbers, Place Value, and Zero After doing many counting activities so that children were familiar with the counting sequence, Ms. Clare asked, "What's your favorite big number?" "Sixty!" "Two hundred!" "A million!" were the cries. " Here's a big number," said Ms. Clare as she formed the number 231 with the place value cards (See figure 1.).

Insert Figure 1

"This is the number 'two hundred thirty one'," said Ms. Clare. Ms. Clare had the children say the number several times together and individually. "I wonder what the number feels like!" said Ms. Clare. "It feels big!" said Sam. "Yes, it does," said Ms. Clare. "Let me show you how it feels." At that point, Ms. Clare stood and demonstrated the number. She represented hundreds with wide circling movements of both arms; tens by forearm movements; and ones by finger movements. "One hundred" (large circling arm movement), "two hundred" (large circling arm movement), "one ten" (forearm movement), "two tens" (forearm movement), "three tens" (forearm movement),"one" (finger-flicking movement). Children then joined Ms. Clare and dramatized the number.

Other numbers were formed with the place value cards and Ms. Clare and the children dramatized them. Each time a number was formed, Ms. Clare had the children chime in and name

the number. Ms. Clare also called attention to how, for example, 231 "feels different" than 123 and other numbers formed from the permutations of the digits, 1, 2 and 3. She then formed two and three-digit numbers with the cards, named them, and had children dramatize them without coaching. After many experiences with forming and dramatizing numbers that did not contain zeros, Ms. Clare had the children watch her dramatize the number 204. She then formed the numbers 204 and 24 with the place value cards and asked the children to identify the number she had dramatized Without hesitation, the children pointed to 204 and said that there were "no tens." Ms. Clare pointed out that the zero was showing in the cards because there were no tens to cover it. When thousands were introduced, Ms. Clare charged the children with figuring out a way to represent this larger amount. Jumping was the first choice for all!

Basic to an understanding of place value is the notion of changes in magnitude, that from right to left in a multi-digit number, each place to the left represents a greater amount. Not only were children literally feeling this difference, but they were also coming to understand that changing the "places" of digits changes the number; thus 123 is different from 321, and 24 is different from 204. Furthermore, by using the place value cards, Ms. Clare had the children see an expanded notation representation of each of the numbers, and recognize the importance of zero as a place holder.

Space and Location. Spatial relationships dealing with location are key to understanding counting, ordinality, symmetry, permutations, and patterns. In the counting sequence, for example, to identify the number that is one more than 5 requires knowledge of what comes immediately "after" 5 or what comes "next." The terms "after, next, before, between" are spatial-temporal ideas. The concept of "between-ness" occurs in counting, ordinal and combinatorial situations. When children are asked to identify counting numbers between 2 and 5, they have to think about numbers they say that are "after" 2 but "before" 5. When challenged with constructing and describing "different" block towers from the same set of blocks, children have to examine the relative positions of the blocks in each tower to tell if the green block is "on top" in one tower or "under" the

blue block in another tower. To complete, continue and generalize patterns, children have to be able to determine where a pattern begins to repeat and what elements come "before" or "after" others.

Preschool children's development of spatial reasoning progresses through two major stages. The first stage is characterized by an egocentric point of view; children use themselves as points of reference for locating positions and orientations of objects in space (Piaget & Inhelder 1956). At the second stage, children are able to relate positions of two objects external to themselves, or two objects in addition to themselves. An important aspect of spatial reasoning is perspective or point of view. Depending on one's point of view, the description of the location of an object may vary. For example, suppose that I am facing a table. The table is "in front" of me when I am facing forward. However, it will be "behind" me after I make a 180-degree turn, or "to my left" or "to my right" if the turn is 90 degrees. If I view six blocks in a row, I could describe one of the blocks as being the "second block from the left" or the "fifth block from the right."

From the previous discussion it can be seen that spatial concepts and language are intimately related. As language and concepts develop, performance on spatial tasks has been shown to improve (Hermer 1994). Thus it is important that young children be given numerous opportunities to develop their spatial and language abilities in tandem. In "Tubes of Cubes," children gain experience in perspective-taking. In "Robot," children develop spatial concepts and language, and some measurement sense, as they physically explore their environment.

Tubes of Cubes To provide opportunities for her children to improve their visual memory, identify the position of objects relative to others, and to learn that designation of position is relative to the point of view of the observer, Ms. Abby played Tubes of Cubes. The activity requires the use of a tube made of clear acetate, the core tube of a roll of paper towels and cubes of different colors. Initially, Ms. Abby identified the colors of three cubes and inserted them into the clear tube, one at a time, saying, "I will put this cube in first. I will put this cube in second. I will put this cube in last." As the children watch, Ms. Abby tilted the tube so that the cube that was inserted

first would emerge first. "Which cube do you think will come out first? second? last?" After children had successfully predicted the emergence of the cubes, Ms. Abby inserted the cubes in the tube, but tilted the tube so that the cubes would emerge from the same end in which they were inserted. Again, she asked the children to predict the color of the cube that would emerge first, second, and last. This time, the cube inserted last emerged first; the cube inserted first emerged last. Once the children were comfortable with the nature of the task, Ms. Abby used the cardboard tube instead of the one made from clear acetate. The children loved the challenge of having to remember the order in which the cubes were inserted and predicting the order in which the they would emerge. The children were delighted when Ms. Abby rotated the filled tube 180 degrees or 360 degrees before tilting the tube and having the children predict the order of the emerging cubes.

Robot Robot (Greenes et al. 1989) is a navigation, path-making and modeling activity. Children give directions to one another to walk from one place to a different place. Initially, a child-size maze might be constructed using pieces of classroom furniture; children direct one another through the maze (Cook et al. 1997). The maze can be modified to give children experience with describing different directions (e.g., forward, backward, right, left), different turn sizes (e.g., big turn, turn a little), and different numbers and sizes of footsteps (e.g., take three baby steps).

Without a maze, children can do the same activity. They can give each other directions for getting from, for example, a water table in the classroom to the door of the classroom. As children gain more experience with giving directions, they become ready to learn ways of keeping track of their routes. One way to help them is to develop a three-dimensional walking-step map (informal scale model) of the classroom. Children can draw or use objects to represent the tables, chairs, book shelves, and other pieces of furniture in the room. The placement of the objects on the map is determined by the number of walking steps required to get from one piece of furniture to another. As children place objects, have them tell how they decided on the locations. Encourage the children to describe the location of each object in relation to some other object on the map (e.g., the sand table is near the door). Once the map is constructed, children can "record" different routes

using different color strings taped to the map. They can also begin to compare the lengths of routes. After experience with the walking maps, children might carry out similar navigation activities on the computer (Clements et al. 1997).

Shape. As children develop, they continually explore objects in their environments. The combination of visual and tactile exploration leads them to insights and conjectures about the nature of objects in their reach or view. Early on, children are capable of distinguishing among circles, squares, and triangles, and between three dimensional objects that roll and those that slide (Gibson 1969). A rich curriculum that builds on these ideas, rather than simply providing additional practice with them, is essential. The curriculum needs to promote observation and investigation of the attributes of both two and three-dimensional shapes, including numbers of sides, corners, edges, and faces; the relative sizes of corners in two-dimensional shapes; the shapes of faces of three dimensional objects; and the symmetries in two-dimensional figures and three-dimensional constructions. Children should be afforded opportunities to identify shapes under various transformations, including reflections and rotations. They can explore the relationship between two and three-dimensional shapes by matching two-dimensional nets to the solids they would cover; by identifying buildings from two-dimensional drawings or photographs of the structures; and by matching shadows cast by objects with the real objects. Ways in which shapes can be composed and decomposed into other shapes can be investigated. As children compose shapes, their attention should be drawn to the kinds of shapes that "fit together" to cover (tessellate) a surface.

During their shape investigations, the opportunity exists to concurrently develop children's deductive reasoning abilities. "Which Kite is Pam's?" and "I'm Thinking of A Block" are activities that foster recognition of shapes and their characteristics, and require children to use clues to eliminate candidates for the solution to a problem.

Which Kite is Pam's? Children are presented with three cards as shown in figure 2. As the teacher reads each clue, the children check the cards, one by one, to see if the picture "matches" the clue. If there is a match, the card remains. If there is no match, the card is removed.

Clues: (1) There is one square on Pam's kite, and (2) there are two triangles on Pam's kite.

Insert Figure 2.

Problems like Pam's Kite can be easily constructed using other shapes and incorporating more number-type clues. The opportunity also exists for children to create their own clues.

I'm Thinking of A Block This activity is similar to Pam's Kite but uses objects rather than pictures. Children have to identify a block from clues about its shape, color or size. To begin, a set of blocks is placed in the children's view, as for example, the large red square, the small red square, the large yellow triangle, and the large green triangle from a set of Attribute Blocks. Children are encouraged to pick up the blocks, "feel" them, and examine them from different perspectives. Once again, as clues are given, one by one, children test each block against the clue. If there is a match, the block is retained; if not, the block is removed. Clues could be: (1) The block I am thinking of is large (small red square is removed); (2) it is a triangle (large red square is removed); and (3) it is yellow (large green triangle is removed). The complexity of this activity is easily modified by changing the number of blocks or the number of clues.

Patterns. Work with patterns involves replication, completion, prediction, extension, and description (generalization). These tasks require children to reason inductively and prepare them for later work with functions and concepts of probability. To accomplish these tasks, children have to identify similarities and differences among elements of a pattern, note the number of elements in the repeatable group, identify when the first group of elements begins to replicate itself, make predictions about what comes next based on given information, and in the case of elements such as body movements, rhythms and pitches, construct a means of recording the pattern for later analysis and replication. Depending on the nature of the elements in the pattern, children may have to apply their knowledge about numbers, shapes, and space and location. For this reason, patterns are a means of integrating concepts from these other domains. The patterns described below

involve kinesthetic, tactile, visual, or auditory stimuli. Solving problems in different contexts with different stimuli helps to build robust understanding of the concepts related to patterns. Patterns involving color, shape, number and texture are not included in the examples that follow because of their prevalence in existing preschool curricula.

Movement Patterns Among the most popular patterns with preschoolers are those that involve physical activity. Children have to interpret visual stimuli and coordinate their body movements to match those of the demonstrator. So, for example, they might begin by imitating the sequence of moves: hop, hop, jump, hop, hop, jump,... or stand, sit, stand, sit,... . Once the pattern has been replicated correctly, children can predict the movement that will come next. Following prediction and continuation, children can be asked to describe the pattern, as for example, "2 hops, 1 jump, 2 hops, 1 jump, 2 hops,...," and then to generalize by recognizing that two hops is always followed by (or comes before) one jump. Since movement patterns have no visual record, there is no way to talk about or compare them at a later time. For this reason, developing a representation or picture of the pattern is useful. For example, the hop-jump pattern might be represented as - - = - - = - - = where each single dash represents a hop (one foot), and the double dash represents a jump (two feet). To be sure that children understand this representation, they should follow it and do the appropriate hops and jumps. Later, the reverse approach can be taken. An hop-jump pattern with dashes can be displayed and the children can try to replicate it with movements.

At a more advanced level, children sit in a circle, and a pattern is established, as for example, clap, wave, tap the floor, clap, wave, tap the floor,.... Individual children are then asked to predict: "Will you always clap/wave/tap the floor? Can you predict what (name of child) will do? How did you know that?"

Rhythmic Patterns Another type of pattern is one created by clapping or clicking sticks, a rhythmic pattern. Analysis of this type of pattern requires interpretation of auditory as well as visual stimuli. For these reasons, rhythmic patterns are complex. As with the hop-jump pattern,

rhythmic patterns require a system of record keeping if they are to be recalled and analyzed at a later time. One way to do this is to use blocks to represent beats, and distances between blocks to represent time intervals between the beats. For a more sophisticated representation, musical notes of different duration (whole, half, quarter, and eighth notes), but without the staff, could be used.

Pitch Patterns Ms. Clare reinforced and expanded children's understanding of patterns by using the context of music. Children listened to her sing a sequence of staccato high and low notes. All high notes had the same pitch; all low notes had the same pitch. A pitch pattern might be: high, high, low, low, high, high, low, ... The children sang along with Ms. Clare. Then they sang on their own. To help the children record the pitch pattern, Ms. Clare drew two horizontal bars on the board. As she sang each note, she recorded a dot above the top bar if the note was high and below the bottom bar if the note was low. Dots were recorded from left to right to show sequence. After children had moved back and forth between recording their singing patterns and singing their recorded patterns, Ms. Clare introduced notes of different duration. Now, in addition to staccato notes, there were sustained notes. Staccato notes were represented by dots and sustained notes represented by dashes. (See figure 3).

Insert Figure 3

Not only did children imitate, complete, extend and describe these patterns, but they also created their own patterns. Some of the creations were sung; others were recorded using the dots and dashes. A most unusual discussion occurred when the children wanted to figure out what notes might belong between the bars and they couldn't decide which note (the concept of between-ness) was best! Although they never reached consensus, they did want to create patterns that used these middle notes. Although auditory patterns are very difficult because they require recall of pitch relationships, the children were delighted with their abilities to, as they said, "do the hard ones."

Measurement. As with pattern, measurement provides a natural context for the integration and application of number, shape, space and location concepts. When children compare lengths, weights and capacities, they use numbers to tell how long or tall, how heavy, or how much. Shape concepts come into play when children investigate sizes of different objects, capacities of containers of different configurations, and weights of objects that vary by substance. Space and location become important when children "line up" objects for comparison, or make comparisons by viewing objects from different perspectives. Critical to developing understanding of concepts of measurement is familiarity with the language used to describe measurement relationships --longer, taller, shorter, the same length; heavier, lighter, the same weight; holds more, holds less, holds the same amount. Children should also be afforded opportunities to compare time intervals.

Length Explorations The construction of towers and trains from blocks or other objects collected throughout the year (hereafter referred to as "junk"), and the comparison of their heights and lengths, provide fine opportunities for the development of children's understanding of linear measures. Charging children to use the blocks or junk objects to make the "tallest" tower or the "longest" train requires them to make decisions about the locations of the objects in relation to one another. Questioning children about their decisions helps them focus their attention on various aspects of the measurement activity and the key concepts. "Why did you put that (point to object) on the bottom of your tower?" "Can you make your tower taller? How would you do that?" Comparing two towers, ask: "How can you tell who has the taller tower? Can you think of another way of telling? Can you make this tower (point to the shorter/longer tower) the same height as that tower? How will you do it?"

Constructing trains from junk, children can make length comparisons like those they do with the towers. With both towers and trains, encourage children to think about the relationship between number of objects and height of tower or length of train. Depending on the sizes of the junk items, a taller tower may be constructed from fewer items than a shorter tower. This type of "conflict"

between number and length provides the stimulus for a rich investigation and discussion of the inverse relationship -- longer objects, fewer number of objects.

Many young children have difficulty understanding how height and length are related. By having children construct towers from cubes that link together and then "laying the towers down" to form trains, the height-length relationship can be clarified. By rotating, reflecting, and translating the linked cubes, children can also see that length is conserved under these various transformations. By using linking cubes of different colors and constructing different arrangements of the same set of cubes, children can see that height and length are conserved when the order of objects is varied or commuted.

Capacity and Weight Preschool classrooms generally have water or sand tables with lots of different shape containers. These are excellent environments for investigation of capacity relationships. In all experiments, containers and objects of different shapes and sizes should be used.

Experiments to compare weights of objects should also incorporate objects of various sizes, substances, and shapes. Initially, using two objects with an easily distinguishable weight difference, as for example a blown-up balloon and a sneaker, children can imitate a two-pan balance scale, holding one object in each hand, identifying which of the two objects is heavier, and, imitating the scale by leaning in the direction of the heavier object. Subsequent activities might involve objects whose weights are not as easily distinguishable, thus promoting the need for a real balance scale. The physical imitation of a two-pan scale aids children with their interpretation of an actual scale.

Time Time explorations for young children should focus on comparisons of amounts of time required to complete various tasks and the terminology necessary to express those comparisons -- more time, less time, the same amount of time. Children should also learn ways to describe duration --long time, short time. Finally, children should be given activities that involve temporal sequencing; determining what happened first, second, and last, and perhaps constructing an event

line (time line) that not only illustrates the sequence of events, but also shows the relative lengths of times of the events. It should be noted that the language associated with making comparisons and describing duration may be difficult for some children because the terminology is the same as that for number and length, respectively.

Seriation This is the process of ordering or creating a sequence by some specific attribute, as for example by length, height, capacity, weight, or amount of time. To accomplish this task, children have to observe and distinguish slight variations of the attribute and order the objects according to a graduated sequence. Initially, two or three objects with great differences in the specified attribute should be compared and ordered. Thereafter, as children become more facile with the task, differences among objects can be diminished and the number of objects can be increased.

Permutations. Permutations, like patterns and measurement, require understanding of concepts from other content domains. Consider the case of comparing block towers made from the same sets of colored blocks. Recognizing an "arrangement" of the set of blocks requires identification of both the distinguishing characteristics of each object, which in this case are color and relative spatial positions of the blocks (e.g., the red block is "on top" of the blue block). To determine if one tower is a permutation of another requires a "matching" of blocks in a one-to-one correspondence. One-to-one correspondence is a fundamental number idea. The matching concept of enumeration does not consider the order in which objects are counted; what does matter is merely that an object is not counted twice. By contrast, the matching concept here is quite different --order is critical.

When children were dramatizing large numbers, as described earlier in this paper, they were feeling the difference between 231 and its various permutations (123, 132, 213, 312, and 321). When children construct towers, they may observe that when the order in which the blocks are arranged changes, a different-looking tower is created.

When exploring permutations using tower building as the activity, you might have children use multiple sets of blocks (e.g., six sets with each set containing three blocks, one red, one green and one blue) and construct the different towers that can be made from those blocks. In this way, children can compare newly constructed towers with existing towers to tell if they are different. The use of the term "different" may cause difficulties for some children. (Cook et al. 1997). They may think that by using different (new) blocks each time, that the towers are different. These children may benefit from coloring a drawing that shows six stacks of three blocks on the same sheet of paper. In this way the meaning of "different arrangement" may be clarified. Later on, the drawing can provide a means of recording, for comparison purposes, the different block towers that can be made from one set of blocks. Although young children may not be able to describe a systematic approach for identifying all of the permutations of a set of blocks, their methods of construction of successive towers may provide insight into the use of such an approach. When this occurs, verbalize your own reasoning as a model for the children to follow.

Representation and Communication

Mathematical relationships can be displayed in a variety of ways. Representations may be pictorial (e.g., drawings, timelines, maps), graphical (e.g., bar graphs made from stacked objects, pictographs), or symbolic (e.g., tables, prose descriptions). When information is displayed, it becomes available for later analysis and discussion. The displays often help to make mathematical relationships more obvious. For many of the activities described earlier that don't lend themselves to visual denotation, as for example, movement and pitch patterns, children need to construct representations in order to be able to talk about the patterns, continue and generalize the patterns, and compare them to other patterns. These records may be in the form of drawings, models (e.g., the use of chips to represent classroom furniture), symbolic representations (e.g., the use of dots and dashes to represent staccato and sustained sounds, respectively), photographs, audio or video tapes.

External representations like those identified above often help children develop mental models or internal representations. Such internal representations embody concepts and relations that enable children to, for example, consider various candidates for the solution to a problem and eliminate others; to select among several solution methods before attempting any; and to make inferences and conjectures.

Encouraging children to talk about their observations, their conjectures, their internal representations, their experiments, their solution plans, and their reasoning strategies, helps them to develop their communication skills, their facility with the language of mathematics, and their awareness of their own mental systems and abilities to communicate with others. It is important to remember that children's language and actions are the windows into their thinking; they are the means by which we can assess children's depths of understanding.

The Role of The Teacher

The teacher is key to the development of young children's mathematical abilities. She is the architect of the environment, the guide and mentor for the explorations, the model reasoner and communicator, and the on-the-spot evaluator of children's performances (Greenes, 1995). She must load the environment so that children bump into interesting mathematics at every turn. To create such an environment, the prior experiences and current interests of children must be identified, capitalized upon, and extended. Explorations should be constructed and carried out in memorable contexts so that the concepts, skills and strategies developed in those contexts can be recalled and transferred more easily to other contexts. Necessary materials that aid children in their investigations have to be readily accessible. Until children become confident to work on their own, the teacher, after introducing activities, identifying goals, and assisting with the planning of what to do, should co-investigate with the children.

For all activities, sufficient time must be provided for children to explore and wrestle with the mathematical ideas. It is during this time, through observations of children's performances and analysis of children's questions and their responses to questions posed to them, that the teacher can

assess their understanding of the key ideas and provide appropriate intervention and guidance. Questions should be used to prompt children to think aloud ("What are you doing now?"), to reflect on their actions ("What did you do before that worked?" " How did you figure that out?"), to make predictions ("What will happen if?" "What will happen next?") , and provide justifications for their choices (Why did you choose that?"). At the end of each investigation or activity, opportunities should be provided for children to share their results, methods and findings with the whole group. By verbalizing her own reasoning in discussions, the teacher serves as a model for the children to emulate.

Conclusion

Research studies in cognition and mathematics education have demonstrated that young children are capable of more complex mathematical thinking than previously thought. Until recently, however, few attempts have been made to capitalize on these talents and provide a stimulating and robust mathematical environment for young children, children who are ready and eager to learn.

To facilitate young children's mathematical development, the preschool mathematics curriculum should change from a collection of unrelated activities, to a cohesive development of the important ideas of number, space and location, shape, patterns, measurement, and permutations. Explorations should be designed to relate to children's daily experiences and current interests, and stimulate them to make conjectures about observations, to make decisions about what to do and how to do it, to create ways of recording their experiments, reflect on their results, and to talk about their methods and conclusions.

The role of the preschool teacher is multifaceted. She must know mathematics well and know what her children know about mathematics. She must construct learning environments to provoke children's curiosity. She must investigate with the children and monitor their performances. She must set the example for oral communication and for the investigation process.

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